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# Calibration and reconstruction in time-series of strain signal of gravitational wave detector

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In operation of laser interferometric gravitational wave detectors, we have to reconstruct the strain signal of space-time metric from an output of electric signal of interferometer servo. In many past works, the strain signal reconstructions have done in frequency domain using Fast Fourier Transform with finite time slices using calibrated transfer function. However optimal length of time slices is different according to methods of gravitational wave searches, e.g. matched filtering, excess peak, etc. Thus we need re-generate calibrated data for all variations of data analysis in this method. Also for more general-purpose data for analysis or for any other evaluation, we develop reconstruction method of strain-calibrated data using time series filter. We will show an analytical framework of the time series filter for the calibration and will demonstrate the numerical processing using CLIO observational data. We apply this process using FIR filter and obtain result that residual error between our new method and previous one is about 20%. The calibration error in the compact binary coalescence is estimated as ~8% large average and ~24% fluctuation in a comparison with ideal waveform.

**KEYWORDS:** Gravitational Wave, Calibration

## 1. Calibration in time domain

The conversion from voltage signals to strain signals in frequency domain is given as

$$h(f) = G(f) \times v(f), \tag{1}$$

where $h(f)$ is spectrum of strain, $v(f)$ is voltage output of an interferometer servo and $G(f)$ is transfer function between strain and voltage. However we cannot generate time series strain signal $h(t)$, with a jointing there finite length. Therefore, we must develop alternative conversion method without FFT to get seamless time series strain signal.

Time series strain signal $h(t)$ can be represented by the convolution in time domain as

$$h(t) = \int_{-\infty}^{\infty} g(t-\tau) \times v(\tau) \mathrm{d}\tau, \tag{2}$$

where $v(t)$ is time series signal of voltage and $g(t)$ is impulse response that gives inverse Fourier Transform of $G(f)$. Actually, calibration in time domain is applied in [1] [2] [3] [4].

We rewrite Eq.(1) for discrete and infinite length data as

$$h(t_i) = 2 \sum_{j=i}^{i+N} g(t_{i+N-1} - t_j) \times v(t_j) \quad (i = 0,\ 1,\ 2, ...,\ \ j = 0,\ 1,\ 2, ...), \tag{3}$$

where factor 2 appear for integrating with negative time only.

## 2. Filter generation for discrete data

A typical laser interferometer type GW detectors has large dynamic range of sensitivity curve. In a design of time series filter g(t), the low frequency cut-off (e.g. 1Hz, 10Hz) is needed to keep its numerical dynamic range.

To reproduce the waveform of a certain frequency, we need filter length of time longer than at least its period. For instance, the wave of 1Hz, namely 1sec period, we need more than 1sec length of filter. Considering with computational cost, it is to be desired that filter length is as short as possible.

To apply these filter processing calibration, we use real data of CLIO that is cryogenic interferometric GW detector in Kamioka. We set filter parameters the following for CLIO data from problem of dynamic range and periodicity;

**1)** Low frequency cutoff: 1Hz (0.1Hz cutoff doesn't satisfy "3)". )
**2)** Filter length: 4sec
**3)** Signal dynamic range: ~ 140dB

We also consider a DC component of time series filter $g(t)$. In general, generated filters have non-zero DC component because of low and high frequency cutoff. We remove DC trend between $t_{\mathrm{start}}$ and $t_{\mathrm{end}}$ as below

$$g'(t) = g(t) - \left( \frac{g(t_{\mathrm{end}}) - g(t_{\mathrm{start}})}{t_{\mathrm{length}}} \right) t. \tag{4}$$

We show transfer function of CLIO $G(f)$ in Fig.1 and generated time series filter $g'(t)$ in Fig.2.

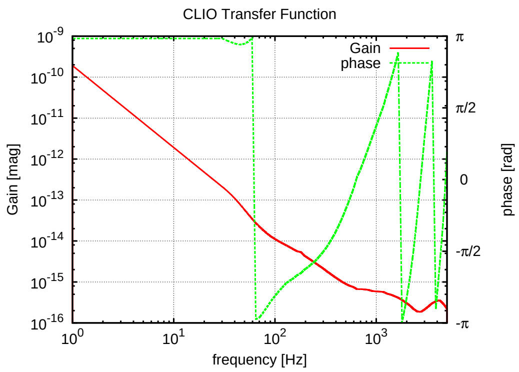


**Fig. 1.** Transfer function of CLIO detector. Red line is gain and green is phase.

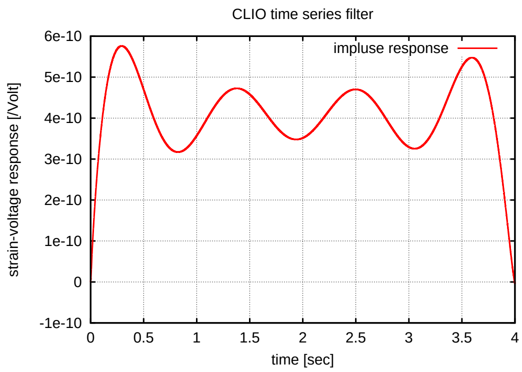


**Fig. 2.** Time series filter for reconstructing strain signal made of transfer function.

## 3. Calibration result

### *3.1 Strain signal by time domain Calibration*

We construct strain signal $h(t)$ of CLIO according to Eq.(3) using filter shown in Fig.2. Fig.3 shows constructed signal $h(t)$. However, since the data used this paper is not of a science run, it is not best sensitivity shown in [5]. We also display spectrum of this signal with that of signal reconstructed in frequency domain in Fig.4.

To compare our new method and previous method, we employ a spectrum of constructed signal in time domain $h_t(f)$ and that in frequency domain $h_f(f)$. We defined a residual of the signal to another as

$$h_{\mathrm{res}}(f) \equiv \frac{|h_t(f)| - |h_f(f)|}{|h_f(f)|}. \tag{5}$$

In this work, we didn't consider error of phase by difficulty of comparing. We use different time by filter length in time domain and frequency domain methods. For example, when we want to get spectrum of 16 sec using 4 sec filter, although we use 16 sec voltage signal in frequency domain, we use 20 sec voltage signal. Fig.5 shows result that is applied Eq.(5) to CLIO signal and we display also average of $h_{\mathrm{res}}(f)$ during about 10min. in Fig.6. We get result that residual error is about 20% at 10 and 100 Hz.These errors larger than previous works and caused from a complexity of transfer function. Transfer function of calibration include that of interferometer, electric circuits and digital feedback filter etc. Previous works measured partial transfer functions that between GW and error signal or that between feedback signal and actuation coupling independently. It is simpler than our method and less errors on calculation. But we could record only transfer function that include all transfer functions. So we can improve this method to prepare the system record individual transfer function. Moreover, we will take into account phase error in next CLIO run by doing hard injection test.

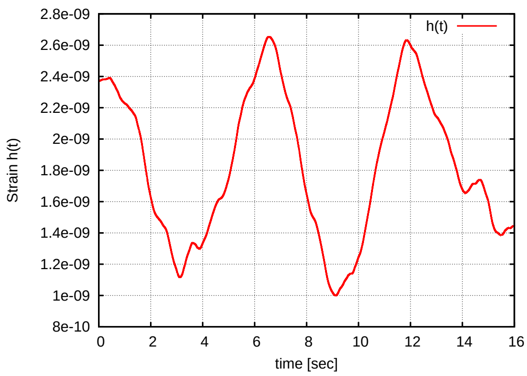


**Fig. 3.** Strain signal of CLIO detector generated by filter process.

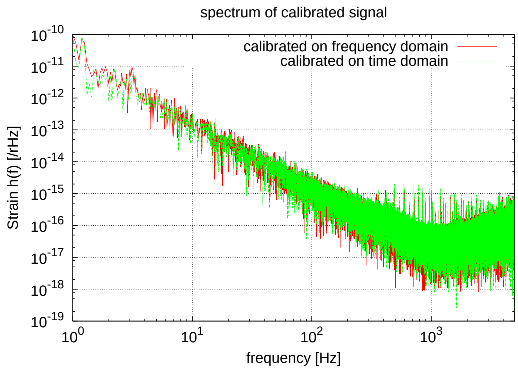


**Fig. 4.** Spectrum of CLIO detector. Red line shows spectrum of time domain process and green is one of frequency domain process.

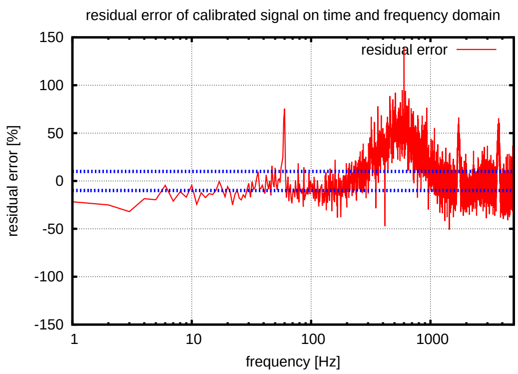


**Fig. 5.** Residual error of spectrum between in time domain and in frequency domain that is defined as Eq.(5).

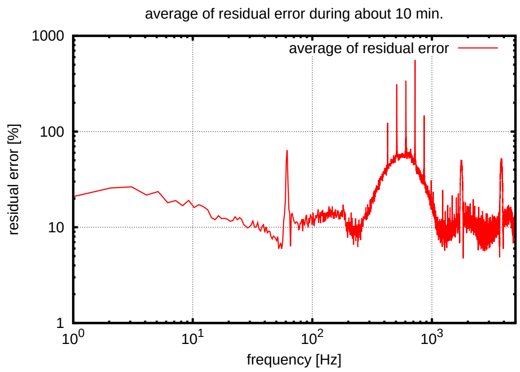


**Fig. 6.** Average of residual error of 16sec spectrum during about 10min.

*3.2 Accuracy of reconstructed gravitational wave signal in the case of compact binary coalescence*

Assuming that Signal-to-noise ratio by Matched filter for certain GW waveform shift from actual signal by our calibration method, we estimate minimal match by our method using

$$(h_{\rm gw}(f),\ h_{\rm gw}(f)) \equiv \int_{-\infty}^{\infty} \frac{h_{\rm gw}(f)h^*_{\rm gw}(f)}{S_n(f)}{\rm d}f \tag{6}$$

where $h_{\rm gw}(f)$ and $h^*_{\rm gw}(f)$ are complex conjugate of spectrum of GW signal. For typical case of GW detection, we suppose 1.4-1.4$M_\odot$ and $\iota = 0$ neutron stars binary system, given by Eq(4.34)-(4.37) in [6] and $S_n(f)$ defined as $|n(f)|^2$. When we reconstruct strain signal $h'_{\rm gw}(f)$, error of transfer function influences $h'_{\rm gw}(f)$ as

$$\begin{aligned} h'_{\rm gw}(f) &= G'(f)\, v_{\rm gw}(f) \\ &= G'(f)\left(G^{-1}(f)\, h_{\rm gw}(f)\right) \\ G(f) &= A(f)\, {\rm e}^{i\phi(f)} \\ G'(f) &= (A(f) + \Delta A(f))\, {\rm e}^{i(\phi(f)+\Delta\phi(f))} \end{aligned} \tag{7}$$

In this time, we ignore the error of phase $\Delta\phi(f)$ and consider error of amplitude $\Delta A(f)$ only. So we assume error of amplitude is equal to $|h_{\rm t}(f)|/|h_{\rm f}(f)|$ and describe as

$$h'_{\rm gw}(f) \equiv (h_{\rm res}(f) + 1)h_{\rm gw}(f) \tag{8}$$

where $h_{\rm res}(f)$ is the value which is shown in the Fig.5.

We calculate

$$\alpha \equiv \frac{(h'_{\rm gw}(f),\ h_{\rm gw}(f))}{(h_{\rm gw}(f),\ h_{\rm gw}(f))} \tag{9}$$

and its distribution is as shown in Fig.7. The average of $\alpha \sim 1.08$ and fluctuation $\sigma/\mu \sim 24\%$ where $\sigma$ and $\mu$ are standard deviation and mean of $\alpha$

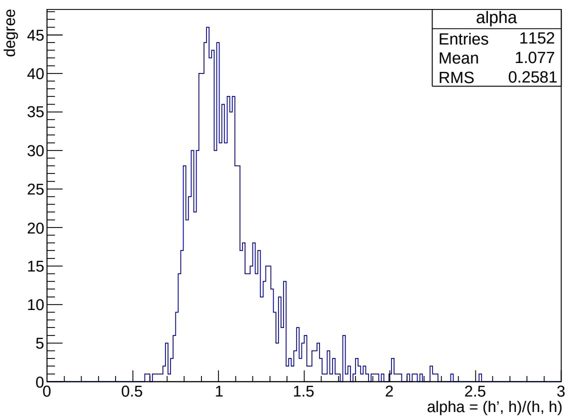


**Fig. 7.** distribution of $\alpha$

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